WEEKLY TEST MEDICAL PLUS -02 TEST - 08 Balliwala SOLUTION Date 25-08-2019

[PHYSICS]

1. The co-ordinates of 3 vertices are given by:

$A:(1,3),\quad B:(2,-4),\quad C:(x_3,y_3)$

We know that for a triangular plate the centre of mass lies at the centroid of the triangle.

$$\therefore (X_{CM}, Y_{CM}) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{1+2+x_3}{3}, \frac{3-4+y_3}{3}\right)$$

But it is given that;

$$(X_{CM}, Y_{CM}) = (0,0)$$

$\therefore \quad \frac{3+x_3}{3} = 0 \quad \text{or} \quad x_3 = -3$

and $\quad \frac{-1+y_3}{3} = 0 \quad \text{or} \quad y_3 = 1$

Hence, $\quad (x_3, y_3) = (-3, 1)$.

2. The co-ordinates of the corners of the square are (0, 0), (2, 0), (2, 2), (0, 2). Hence,

$$X_{CM} = \frac{m_1x_1+m_2x_2+m_3x_3+m_4x_4}{m_1+m_2+m_3+m_4}$$

$$= \frac{2\times0+3\times2+5\times2+8\times0}{2+3+5+8} = \frac{16}{18} = \frac{8}{9}\text{ m}$$

$$Y_{CM} = \frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4}{m_1+m_2+m_3+m_4}$$

$$= \frac{2\times0+3\times0+5\times2+8\times2}{2+3+5+8} = \frac{26}{18} = \frac{13}{9}\text{ m}$$

$\therefore$ Co-ordinates of the centre of mass $= \left(\frac{8}{9}, \frac{13}{9}\right)$.

3. Mass of the disc removed $= \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$

Remaining mass $= M - \frac{M}{4} = \frac{3M}{4}$

Let the origin of the co-ordinate system coincide with the centre of mass of whole disc. Now, we know that;

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

X_{CM} will be zero, when

$$m_2 x_2 = -m_1 x_1$$

$\therefore \quad x_2 = -\frac{m_1}{m_2} x_1$

Here, $\quad m_1 = \frac{M}{4}, \quad x_1 = \frac{R}{2}$

and $\quad m_2 = \frac{3M}{4}$ (for remaining mass)

Hence, $\quad x_2 = -\frac{M/4}{3M/4} \cdot \frac{R}{2} = \frac{-R}{6}$

i.e., $\frac{R}{6}$ from the centre (on LHS).

4. Mass of the disc removed $= \frac{M}{\pi (28)^2} \times \pi (21)^2 = \frac{9M}{16}$

Remaining mass $= \frac{7M}{16}$

Using the same method as followed in the above question,

$$\frac{7M}{16} \times OO_2 = \frac{9M}{16} \times OO_1$$

As, $\quad OO_1 = (28 - 21)$ cm $= 7$ cm

Hence, $\quad OO_2 = \frac{9}{7} \times 7$ cm $= 9$ cm.

5. Let the circular disc of radius a be made up of the circular section of radius b and remainder. Further let the line of symmetry joining the centres O and O_1 be the x-axis with O as origin. The centre of mass of the disc of radius a will be given by:

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \ldots(i)$$

while Y_{CM} and Z_{CM} will be zero (as for all points on x-axis, y and $z = 0$)

If σ be the density of the material of disc,

$m_1 = \pi b^2 \sigma$ and $x_1 = c$

$m_2 = \pi(a^2 - b^2)\sigma$ and $x_2 = ?$

$M = (m_1 + m_2) = \pi a^2 \sigma$ and $X_{CM} = 0$

From eqn. (i),

$$0 = \frac{\pi b^2 \sigma (c) + \pi(a^2 - b^2)\sigma x_2}{\pi a^2 \sigma}$$

$\therefore \quad x_2 = \frac{-cb^2}{(a^2 - b^2)}$

i.e., the centre of mass of the remainder (say O_2) is at a distance $cb^2/(a^2 - b^2)$ to the left of O on the line joining the centres O and O_1.

6. Let ρ be the density of lead.

Then, $M = \frac{4}{3}\pi R^3 \rho$ = mass of total sphere

$$m_1 = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \text{mass of removed part} = \frac{M}{8}$$

$$m_2 = M - \frac{M}{8} = \frac{7M}{8} = \text{mass of remaining sphere}$$

Choosing the centre of big sphere as the origin,

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

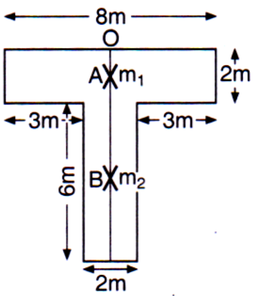
$$0 = \frac{\frac{M}{8} \times \frac{R}{2} + \frac{7M}{8} \times x_2}{M}$$

Solving, we get; $x_2 = \frac{-R}{14}$

i.e., centre of mass of hollowed sphere would be at a distance of $R/14$ on left of O,

i.e., shift in centre of mass $= \frac{R}{14}$.

7. Taking parts A and B as two bodies of same system,



$$m_1 = l \times b \times \sigma = 8 \times 2 \times \sigma = 16\sigma$$

$$m_2 = l \times b \times \sigma = 6 \times 2 \times \sigma = 12\sigma$$

Choosing O as the origin,

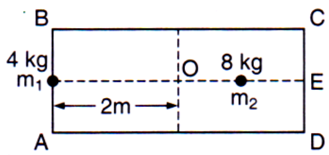
$$x_1 = 1 \text{ m}, \quad x_2 = 2 + 3 = 5 \text{ m}$$

$$\therefore \quad X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{16\sigma \times 1 + 12\sigma \times 5}{16\sigma + 12\sigma} = \frac{19}{7}$$

$$= 2.7 \text{ m from } O.$$

8. Here, $m_1 = 4$ kg; $x_1 = 2$ m



$$m_2 = 8 \text{ kg}; \quad x_2 = ?$$

$$X_{CM} = 0$$

$$\therefore \quad X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

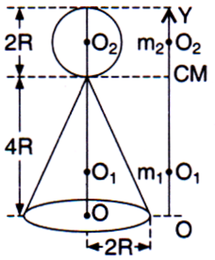
$$0 = \frac{4 \times 2 + 8x_2}{4 + 8}$$

$$\therefore \quad x_2 = -\frac{8}{8} = -1 \text{ m}.$$

$\therefore$ m_2 (= 8 kg) must be placed at 1 m from O on OE.

9. If the density of cone be ρ, then its mass will be,

$$m_1 = \frac{1}{3}\pi(2R)^2(4R)\rho = \frac{16}{3}\pi R^3\rho$$



and its centre of mass will be at a height

$$\frac{h}{4} = \frac{4R}{4} = R$$

from O on the line of symmetry,

i.e., $y_1 = R$

Similarly, the mass of the sphere,

$$m_2 = \frac{4}{3}\pi R^3(12\rho) = 16\pi R^3\rho = 3\,m_1$$

and its centre of mass will be at its centre O_2, *i.e.*, $y_2 = 4R + R = 5R$ (from O).

Now, treating the sphere and cone as point masses with their masses concentrated at their centres of mass respectively and taking the line of symmetry as y-axis with origin at O, the centre of mass of the toy is given by:

$$Y_{CM} = \frac{m_1\,y_1 + m_2\,y_2}{m_1 + m_2} = \frac{m_1 \times R + 3m_1 \times 5R}{m_1 + 3m_1} = 4R$$

i.e., centre of mass of the toy is at a distance of $4R$ from O on the line of symmetry, *i.e.*, at the apex of the cone.

10. According to the equation of motion of the centre of mass,

$$M\,\vec{a}_{CM} = \vec{F}_{ext.}$$

If $\vec{F}_{ext.} = 0$, $\vec{a}_{CM} = 0$

i.e., $\vec{v}_{CM} = \text{constant}$

i.e., if no external force acts on a system (or resultant external force acting on a system is zero) the velocity of its centre of mass remains constant (*i.e.*, **velocity of the centre of mass is unaffected by internal forces**). Hence, the kinetic energy and momentum of the system also remain constant.

So, if the centre of mass of a system is at rest (or in the state of uniform motion) it will remain at rest (or in the state of uniform motion) unless acted upon by an external force. Thus, if $\vec{F}_{ext.} = 0$, it is possible that the position of the centre of mass may change at a constant rate.

11. As discussed in the above question 25, if $\vec{F}_{ext.} = 0$, the velocity of the centre of mass of the system remains constant and is not affected by internal forces whatever may be their direction of action.

12. The two bodies will move towards their common centre of mass but the location of the centre of mass will remain unchanged, *i.e.*, CM remains at rest w.r.t. A as well as B.

13. Initially, the particles are at rest, so the velocity of the centre of mass,

$$\vec{v}_{CM} = \frac{m_1 \times 0 + m_2 \times 0}{m_1 + m_2} = 0$$

As here $\vec{F}_{ext.} = 0$, so

$$\vec{v}_{CM} = \text{constant}$$

i.e., $\dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = 0$ (at all instants)

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$

or $m_1 \dfrac{\Delta\vec{r}_1}{\Delta t} + m_2 \dfrac{\Delta\vec{r}_2}{\Delta t} = 0$

or $m_1 \Delta\vec{r}_1 + m_2 \Delta\vec{r}_2 = 0$ (as $\Delta t \neq \infty$)

or $m_1\vec{d}_1 + m_2\vec{d}_2 = 0$ (with $\Delta\vec{r} = \vec{d}$)

or $m_1 d_1 - m_2 d_2 = 0$

(as direction of $\vec{d}_2$ is opposite to $\vec{d}_1$)

or $m_1 d_1 = m_2 d_2$

But given that $d_1 + d_2 = d$, so that

$$d_1 = \frac{m_2 d}{m_1 + m_2} \quad \text{and} \quad d_2 = \frac{m_1 d}{m_1 + m_2}$$

Now, as d_1 and d_2 represent the position of the centre of the mass relative to m_1 and m_2 respectively, the particles will collide at the centre of mass of the system.

14. As seen in question 28, two particles will meet at their centre of mass

∴ Distance of the centre of mass from 8 kg mass

$$= \frac{8 \times 0 + 4 \times 12}{8 + 4} = 4 \text{ m}.$$

15.

16. The centre of mass reference frame is one in which the centre of mass is at rest. So, the velocity of the heavier block in this frame just after the kick is,

$$v_1' = v_1 - V_{CM} = 14 - 10 = 4 \text{ m/s}$$

and that of lighter block is,

$$v_2' = v_2 - V_{CM} = 0 - 10 = -10 \text{ m/s}.$$

17. Given that the system is initially at rest,

i.e., $\vec{V}_{CM} = 0$

$\therefore \quad \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = 0$

or $\quad m_1\vec{v}_1 + m_2\vec{v}_2 = 0$

or $\quad m_1 \dfrac{\Delta\vec{r}_1}{\Delta t} + m_2 \dfrac{\Delta\vec{r}_2}{\Delta t} = 0$

or $\quad m_1 \Delta\vec{r}_1 + m_2 \Delta\vec{r}_2 = 0$

Now, here in boat-man system if the man moves towards right the boat moves towards left.

$\therefore \quad m_1 \Delta r_1 = m_2 \Delta r_2 \quad \ldots(i)$

($\because \Delta r_1$ is opposite to Δr_2)

If Δr_2 is the displacement of boat relative to shore, then the displacement of man relative to shore would be $(9 - \Delta r_2)$,

i.e., $\quad \Delta r_1 = 9 - \Delta r_2 \quad \ldots(ii)$

From eqn. (i) and (ii),

$m_1(9 - \Delta r_2) = m_2 \Delta r_2$

or $\quad 100(9 - \Delta r_2) = 500 \Delta r_2$

$\therefore \quad \Delta r_2 = \dfrac{100 \times 9}{600} = 1.5 \text{ m}$

i.e., Boat moves 1.5 m relative to shore in the direction opposite to the displacement of the man.

18. Given that initially the system is at rest, so

$$\vec{V}_{CM} = 0$$

Now, as in the motion of dog no external force is applied to the system, hence

$$\vec{V}_{CM} = \text{constant} = 0$$

i.e., $\quad \dfrac{m\vec{v}_1 + M\vec{v}_2}{m + M} = 0$

or $\quad m\vec{v}_1 + M\vec{v}_2 = 0 \quad$ [as $(m + M)$ = finite]

or $\quad m \dfrac{\Delta\vec{r}_1}{\Delta t} + M \dfrac{\Delta\vec{r}_2}{\Delta t} = 0$

or $\quad m\Delta\vec{r}_1 + M\Delta\vec{r}_2 = 0 \quad$ (as $\Delta\vec{r} = \vec{d}$ = displacement)

or $\quad md_1 - Md_2 = 0 \quad$ (as $\vec{d}_2$ is opposite to $\vec{d}_1$)

or $\quad md_1 = Md_2 \quad \ldots(i)$

Now, when dog moves 4 m towards shore relative to boat, the boat will shift a distance d_2 relative to shore opposite to the displacement of dog; so, the displacement of dog relative to shore (towards shore) will be,

$d_1 = 4 - d_2 \quad \ldots(ii)$

(*i.e.*, $d_1 + d_2 = d_{rel.} = 4$).

Putting the value of d_2 from eqn. (ii) in eqn. (i),

$md_1 = M(4 - d_1)$

or $\quad d_1 = \dfrac{M \times 4}{m + M} = \dfrac{20 \times 4}{5 + 20} = 3.2 \text{ m}$

As initially the dog was 10 m from the shore, so now he will be $(10 - 3.2) = 6.8$ m from the shore.

19. Given that initially the system is at rest,

i.e., $\vec{V}_{CM} = 0$

so $\vec{V}_{CM} = \text{constant} = 0$

i.e., $\dfrac{m\vec{v} + M\vec{V}}{m + M} = 0$

or $m\vec{v} + M\vec{V} = 0$ [as $(m + M) = \text{finite}$]

i.e., $M\vec{V} = -m\vec{v}$ ...(i)

Furthermore, here it is given that;

$\vec{v}_{rel.} = \vec{v} - \vec{V}$ ...(ii)

Putting the value of $\vec{v}$ from eqn. (ii) in eqn. (i), we get;

$M\vec{V} = -m(\vec{v}_{rel.} + \vec{V})$

or $\vec{V} = -\dfrac{m\vec{v}_{rel.}}{(m + M)}$ ...(iii)

Thus, it is clear that the direction of motion of balloon is opposite to that of climbing ($\vec{v}_{rel.}$), *i.e.*, vertically down.

20. From eqn. (i) of question. 19 we find that

$m\vec{v} + M\vec{V} = 0$

$m\dfrac{\Delta\vec{r}_1}{\Delta t} + M\dfrac{\Delta\vec{r}_2}{\Delta t} = 0$

or $m\,\Delta\vec{r}_1 + M\,\Delta\vec{r}_2 = 0$

or $m\,\vec{d}_1 + M\,\vec{d}_2 = 0$ [$\because \Delta\vec{r} = \vec{d}$]

or $md_1 - Md_2 = 0$

or $md_1 = Md_2$ ...(iv)

Now, as the man climbs up L towards the balloon (relative to balloon), the balloon will descend a distance d_2 downwards relative to the ground, so that upward displacement of man relative to ground will be,

$d_1 = L - d_2$ (*i.e.*, $d_1 + d_2 = L$) ...(v)

Putting the value of d_1 from eqn. (v) in eqn. (iv), we get;

$m(L - d_2) = Md_2$

i.e., $d_2 = \dfrac{mL}{m + M}$ ...(vi)

i.e., balloon will descend by $\dfrac{mL}{m + M}$ relative to ground when the man climbs up a distance L (relative to balloon).

21. When man stops climbing,

$\vec{v}_{rel.} = 0$

so that from eqn. (iii) of question 34, $\vec{V} = 0$, *i.e.*, balloon will also stop descending and will become stationary relative to the ground.

22. Refer to question 13 Two particles collide at their centre of mass.
$\therefore$ Distance of CM from P

$$= \frac{0.1 \times 0 + 0.3 \times 1}{0.1 + 0.3} = 0.75 \text{ m}.$$

23. Consider P: $s_1 = \frac{1}{2} a_1 t^2$

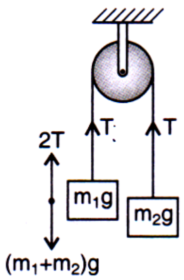
or $0.75 = \frac{1}{2}\frac{F}{m_1} t^2 = \frac{1}{2} \times \frac{10^{-2}}{0.1} \times t^2$

$\therefore$ $t = \sqrt{15}$ sec

24. Under mutual attraction, the centre of mass remains at rest *i.e.*, velocity is zero.

25. The equation of motion of the centre of mass is,

$$M\,\vec{a}_{CM} = \vec{F}_{ext.}$$



And as there is no external force in horizontal direction, so the centre of mass of the system does not change along horizontal direction.
For vertical motion of the centre of mass,

$$(a_{CM})_y = \frac{F_{ext.}}{M} = \frac{(m_1 + m_2)g - 2T}{(m_1 + m_2)} \quad \ldots(i)$$

Further, $a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

$$= \frac{m_1 - m_2}{m_1 + m_2} a \qquad [\because \vec{a}_1 = a \text{ and } \vec{a}_2 = -a]$$

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g \qquad \left[\because a = \frac{m_1 - m_2}{m_1 + m_2} g\right]$$

However, the equations of motion of two blocks are

$$m_2 g - T = m_2 a$$
$$T - m_1 g = m_1 a$$

Eliminating a, we get;

$$T = \frac{2m_1 m_2}{m_1 + m_2} g \quad \ldots(ii)$$

Putting eqn. (ii) in eqn. (i), we get;

$$(a_{CM})_y = \frac{(m_1 + m_2)^2 - 4m_1 m_2}{(m_1 + m_2)^2} g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

[**Note:** It is clear that acceleration of CM is always vertically downward irrespective of whether m_1 is heavier or m_2.]

26. Force F_A on the particle A is given by:

$$F_A = m_A a_A = \frac{m_A v}{t} \quad \ldots(i)$$

Similarly, $F_B = m_B a_B = \frac{m_B \times 2v}{t} \quad \ldots(ii)$

Now, $\frac{m_A v}{t} = \frac{m_B \times 2v}{t} \qquad (\because F_A = F_B)$

So, $m_A = 2m_B$

Hence, the speed of the centre of mass of the system

$$V_{CM} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{2m_B v - m_B \times 2v}{2m_B + m_B} = 0.$$

27. Here, the centre of mass of the system remains unchanged when the mass m moved a distance $L\cos\theta$, let the mass $(m+M)$ moves a distance x in the backward direction.

$\therefore \quad (M+m)x - mL\cos\theta = 0$

$\therefore \quad x = (mL\cos\theta)/(m+M)$.

28. Horizontal distance travelled by the centre of mass before hitting the ground is $R = \dfrac{u^2 \sin 2\theta}{g}$ (since the path of the centre of mass does not change due to the forces of explosion).

29. Since, P is the freely falling body, it hits the ground vertically downwards, *i. e.*, at a distance of $R/2$ from the starting point. Since, the centre of mass hits the ground at a distance R from the starting point, then

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\therefore \quad R = \frac{\frac{m}{2} \times \frac{R}{2} + \frac{m}{2} \times x_2}{m}$$

$$\therefore \quad x_2 = 2R - \frac{R}{2} = \frac{3R}{2}.$$

30. Motion of the centre of mass is exactly similar to that of translatory motion of a body that is thrown into air.

$$u_x = u\cos\theta, \qquad u_y = u\sin\theta$$

$$= \frac{10}{\sqrt{2}}\text{ m/s} \qquad = \frac{10}{\sqrt{2}}\text{ m/s}$$

$$v_x = \frac{10}{\sqrt{2}}\text{ m/s}$$

(since there is no change in the horizontal velocity)

$$v_y^2 - u_y^2 = 2(-g)(h)$$

$$v_y^2 = u_y^2 - 2gh = \frac{100}{2} - 2 \times 10 \times 1 = 30$$

$$\therefore \text{Net velocity of CM} = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{\frac{100}{2} + 30} = \sqrt{80} = 4\sqrt{5}\text{ m/s}.$$

31.

$$F_{ext.} = \sqrt{F_x^2 + F_y^2} = \sqrt{16^2 + 8^2} = 8\sqrt{5}\text{ N}$$

$$M = m_1 + m_2 = 8 + 8 = 16\text{ kg}$$

$$\therefore \quad \vec{F}_{ext.} = M\,\vec{a}_{CM}$$

$$|\vec{a}_{CM}| = \frac{|\vec{F}_{ext.}|}{M} = \frac{8\sqrt{5}}{16} = \frac{\sqrt{5}}{2}\text{ m/sec}^2$$

$\vec{a}_{CM}$ lies in the direction of $\vec{F}_{ext.}$. Angle made by $\vec{F}_{ext.}$ with x-axis

$$= \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{16}{8} = \tan^{-1}(2).$$

32. $$\vec{F}_{ext.} = M\,\vec{a}_{CM}$$

i.e., $\vec{a}_{CM}$ lies in the direction of $\vec{F}_{ext.}$

Here, $\vec{F}_{ext.} = 5(2\hat{i} + 3\hat{j} + 5\hat{k})$

$\vec{a}_{CM} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

Since, $\vec{F}_{ext.}$ and $\vec{a}_{CM}$ are not lying in the same direction, given data is incorrect.

33. $$R_{CM} = \frac{12 \times 0 + 16 \times 1.12 \times 10^{-10}}{12 + 16}$$

$$= \frac{16}{28} \times 1.12 \times 10^{-10}\ \text{m} = 0.64 \times 10^{-10}\ \text{m}.$$

34. $m_1 = 10$ kg, $m_2 = 2$ kg

$\vec{v}_1 = 2\hat{i} - 7\hat{j} + 3\hat{k}$

$\vec{v}_2 = -10\hat{i} + 35\hat{j} - 3\hat{k}$

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$= \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{10 + 2} = 2\hat{k}\ \text{m/s}$$

35. $$X_{CM} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$= \frac{ml + 2m \cdot 2l + 3m \cdot 3l + \ldots}{m + 2m + 3m + \ldots}$$

$$= \frac{ml(1 + 4 + 9 + \ldots)}{m(1 + 2 + 3 + \ldots)}$$

$$= \frac{l\,\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{l(2n+1)}{3}$$

36. $$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$$

$m_1 = m_2 = m$

$a_1 = 0;\quad a_2 = a$

$\therefore\quad a_{CM} = \frac{ma}{2m} = \frac{a}{2}.$

37.

38. We know that as all the metal balls are identical in mass and radius, therefore the centre of mass of the system will be at the point of intersection of their medians.

39. $$a = \frac{3m - m}{3m + m}g = \frac{g}{2}$$

Acceleration of centre of mass

$$= \frac{3m \times \frac{g}{2} - \frac{mg}{2}}{3m + m} = \frac{g}{4}.$$

40. Acceleration of system,

$$a = \frac{mg \sin 60° - mg \sin 30°}{2m}$$

or $$a = \left(\frac{\sqrt{3}-1}{4}\right) g$$

Now, $\vec{a}_{common} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m}$

Here, $\vec{a}_1$ and $\vec{a}_2$ are $\left(\frac{\sqrt{3}-1}{4}\right) g$ at right angles.

Hence, $|\vec{a}_{common}| = \frac{\sqrt{2}}{2} a = \frac{a}{\sqrt{2}} = \left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right) g$.

41. Unless $m_1 = m_3$, the centre of mass of all the four particles can never be at the centre of the square.

42. The rope tension is the same both on the left and right hand side at every instant and consequently momentum of both sides are equal.

$\therefore Mv = (M - m)(-v) + m(v_r - v)$

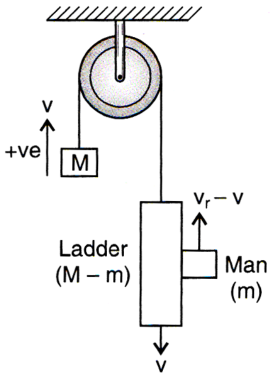
or $v = \frac{m}{2M} v_r$

Momentum of the centre of mass is,

$P = P_1 + P_2$

or $2Mv_{com} = Mv + Mv$

$\therefore$ $v_{com} = v = \frac{m}{2M} v_r$.



43.

$$\vec{v}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$\frac{\vec{v}_1 + \vec{v}_2}{2} = (\hat{i} + \hat{j})\ \text{m/s}$$

Similarly, $\vec{a}_{com} = \frac{\vec{a}_1 + \vec{a}_2}{2} = \frac{3}{2}(\hat{i} + \hat{j})\ \text{m/s}^2$

Since, $\vec{v}_{com}$ is parallel to $\vec{a}_{com}$ the path will be a straight line.

44. If mass is non-uniformly distributed, then centre of mass of ring may lie from origin to circumference. Hence, $0 \le b \le a$.

45. Velocity of man with respect to ground is 1 m/s in opposite direction.

Hence, $v_{com} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{40 \times 2 - 80 \times 1}{40 + 80} = 0$.

[CHEMISTRY]

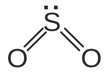
46 CH_2=CH–CH_2–C≡CH has 10σ-bonds are 3π-bonds

47. 34 electrons

48.

49.

50. BF_3 and NO_2^- have sp^2-hybridised central atom while NH_2^- and H_2O have sp^3 hybridised central atom.

51.  sp^2-hybridisation

52. SF_4 and I_3^- and PCl_5 are sp^3d-hybridised. In general $PCl_5(g)$ in considered sp^3d. In solid state, PCl_5 exists as $(PCl_4)^{\oplus}(PCl_6)^{\ominus}$ with sp^3 and sp^3d^2-hybridisations respectively.

$SbCl_5^{2-}$ has 5σ bonds and one lone pair. It is sp^3d^2-hybridised.

53. Four atoms directly related with C≡C are linearly arrnaged

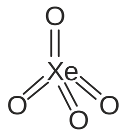
54. XeF has 8 electrons in valence shell. In XeF_2, XeF_4 and XeF_6, two sigma bonds, four sigma bonds and six sigma bonds are respectively formed. Hence, in XeF_2 3 pairs of electrons are left, in XeF_4 2 pairs of electron are left and in XeF_6 only 1 pair of electron is left.

55. In BH_3, B-atom forms 3σ-bonds has sp^2-hybridization. In B_2H_6, each B-atom is joined with 4H atoms and is sp^3-hybridization

56. $\underset{\text{Al is } sp^2}{AlH_3} \xrightarrow{H^-} \underset{\text{Al is } sp^3}{AlH_4^-}$

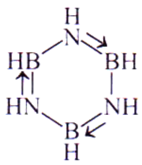
57. Each f C^1 and C^2 are forming two sigma bonds. Hence, both are sp-hybridised.

58.

59.  XeO_4 has 4σ- and 4π-bonds.

60. In SF_4, sulphur atom is sp^3d hybridized with two axial and two equitorial F-atoms and one lone pair on equitorial position.
The axial S–F bonds are larger than equitorial S–F bonds.

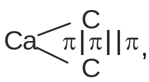
61. In methane C-atom is sp^3-hybridized with 25 s-character. In ethene, it is sp^2 with 33 s-character has to be less than 25 (actual value is 21.43)

62.  12 σ and 3π bonds. Ratio σ : π bonds = 4 : 1

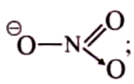
63. Highest product of charges of ions.

64. Phosphorus ($1s^22s^22p^63s^23p^6$ 3d) can expand electronic configuration become of availability of 3d-subshell in valence shell.
Nitrogen ($1s^22s^22p^3$) has no d-subshell in valence shell for expansion of electronic configuration.

65. Cu^{2+} and SO_4^{2-} have coulombic forces of attraction giving rise to ionic bond. Four H_2O molecules form coordinate bonds with Cu^{2+}. One H_2O molecule joins two H_2O related to Cu^{2+} and also SO_4^{2-} by H-bonds. H_2O itself has covalent bonds.

66. , one sigma and two pi bonds

67. x is related to sp^3-hybridized C-atom, y is related to sp^2-hybridized C-atom and z is related to sp-hybridised C-atom.

68. ; bond angle in H_2O is 104.5°

69. In B_2H_6, each BH_3 unit has 6 electrons on B-atom

70. A covalent bond is formed by the partial overlap of electron clouds of half filled orbitals.

71. Highest product of charges of ions.

72. C

73. O_2^- has one unpaired electron $(\pi^*_{2p_y})^1$.

74. L.E. is directly proportional to charge and inversely proportional to size.